Project 3

Annual Personal Consumption of Coffee

Natalia Belén Morales Flores
Math 137 - Spring 2025

Introduction

Coffee consumption is a common habit influenced by several factors, including age and sleep patterns. As a heavy coffee drinker, I am interested in understanding how these factors that affect coffee consumption can help in areas like public health, workplace productivity, etc. Utilizing my statistics skills, I aim to explore this topic of such a personal influence.

In this project, I model coffee consumption as a function of age and average hours of sleep per night. The objective is to find a mathematical model, visualize its behavior, and explore how integrals and Riemann sums can help estimate average coffee consumption over a given age range.

Function

I chose a linear model to estimate coffee consumption based on age (a) and number of hours of sleep (s):

$$C(a, s) = \beta_0 + \beta_1 \cdot a + \beta_2 \cdot s + e$$

where **C(a,s)** is the annual coffee consumption in cups per year, $\boldsymbol{a}$ is the age of the person in years, $\boldsymbol{s}$ is the average hours of sleep per night, $\boldsymbol{\beta_0}$, β_1 and β_2 are coefficients drawn from realistic data, and $\boldsymbol{e}$ is the error term representing random variation. For this project, I will assign values to β_0, β_1, and β_2 based on external research to reduce the number of variables in the function.

β_0 = 150, this is the baseline consumption for people 30 years old or younger according to Balance

Coffee UK.
β_1 = 5, we will assume that for every year a person gets older, they add 5 cups to their yearly consumption. This value can change depending on what the data collection shows.
β_2 = -20, effect of sleep, for every extra hour of sleep, we will assume that the person consumes 20 cups less per year.

Graph

I am ranging the plot for ages 17 to 70, since people younger and older do not consume as much coffee. Hours of sleep are also ranging from 4 to 9, to avoid extreme values and make it as realistic as possible.

In[7]:=
```
b0 = 150; b1 = 5; b2 = -20;
CoffeeConsumption[A_, S_] := b0 + b1 * A + b2 * S

Plot3D[CoffeeConsumption[A, S], {A, 18, 70}, {S, 4, 9},
 AxesLabel → {"Age", "Hours of Sleep", "Coffee Consumption"},
 PlotLabel → "Coffee Consumption as a Function of Age and Sleep",
 ColorFunction → "Rainbow", Mesh → None]
```

Out[9]=

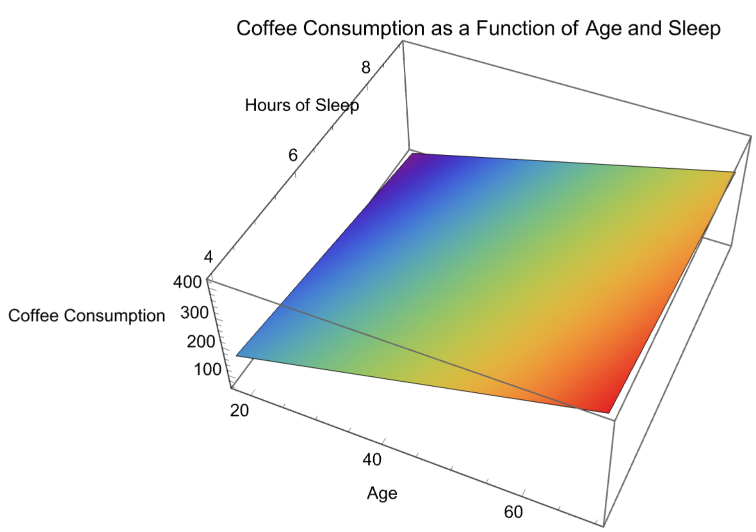

Figure 1. Coffee Consumption as a Function of Age and Sleep

Riemann sums

The concept of accumulation helps us understand the total coffee consumption over a period (here, from age 18 to 70). Integrating the function over a specific range will give the total coffee consumption for a population within that range.

$$\int_{18}^{70} C(a, s)\, d A$$

The integral gives the accumulated coffee consumption over the lifespan.
Now the interval [a,b] is divided into n-equal width subintervals ΔA = $\frac{b-a}{n}$, and we also have the sample points $A_1, A_2, \ldots, A_n$. Now we approximate the integral using the Riemann Sum

$$\int_{a}^{b} C(a, s)\, d A \approx \Delta A * [C(A_1, s) + C(A_2, s) + \ldots + C(A_n, s)]$$

$$\approx \Delta A * [(b_0 + b_1 A_1 + b_2 s) + (b_0 + b_1 A_2 + b_2 s) + \ldots]$$

Specifically with a = 18, b = 70, s = 6, and n = 10:

$$\Delta A = \frac{70-18}{10} = 5.2$$

$$5.2 * [(150 + 5 * 20.6 - 20 * 6) + (150 + 5 * 25.8 - 20 * 6) + \ldots]$$

In this context of modeling coffee consumption, Riemann sums provide a practical way to approximate the total coffee consumption of an individual over a given age range. By dividing the lifespan from age 18 to 70 into equal subintervals, each of width ΔA, and calculating the coffee consumption at specific ages within these subintervals, we can add these values to estimate the total amount of coffee consumed. This approach is valuable because it allows us to understand the concept of accumulation without directly using calculus — instead, we sum the contributions of coffee consumption at each age. Specifically, the Riemann sum tells us that if an individual consumes coffee according to the function C(a,s) = 150 + 5a - 20s, where s = 6 (6 hours of sleep), the sum of these values across subintervals gives an estimate of their total lifetime coffee consumption. This is important because it provides a clear, step-by-step calculation that shows how annual coffee consumption builds up into a lifetime total, making the concept of accumulation intuitive. It also allows us to see how the total changes depending on the number of subintervals. More intervals mean a more accurate estimate, which directly connects this approach to the idea of an integral.

Integrals

Total coffee consumption over a lifetime:

In[29]:=
```
b0 = 150; b1 = 5; b2 = -20;

CoffeeConsumption[A_, S_] := b0 + b1 * A + b2 * S

AccumulatedCoffee[x_] := NIntegrate[CoffeeConsumption[t, 6], {t, 18, x}]

accumulationPlot = Plot[AccumulatedCoffee[x], {x, 18, 70},
   PlotLabel → "Accumulated Coffee Consumption from Age 18",
   AxesLabel → {"Age", "Accumulated Coffee"}, PlotStyle → Thick];
accumulationPlot
```

Out[33]=

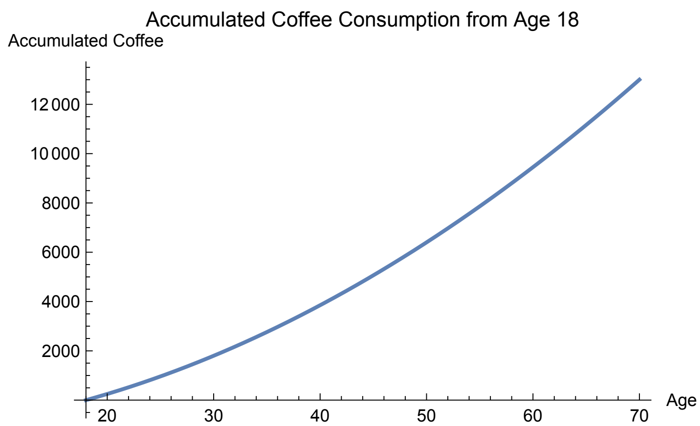

Figure 2. Accumulated Coffee Consumption from Age 18.

The accumulated coffee consumption graph (Figure 2) represents the total number of coffee cups consumed by an individual who sleeps 6 hours per night, starting at age 18 and ending at age 70. The curve is upward sloping and concave, indicating that coffee consumption increases at an increasing rate due to the positive effect of age in the model ($b_1 = +5$). The value of 12,000 cups at age 70 means that, on average, this person consumes around 12,000 cups of coffee over their entire adult life (age 18 to 70). Mathematically, this is the integral of the coffee consumption function from age 18 to 70:

$$\int_{18}^{70} (150 + 5\,A - 20 * 6)\, d\,A = 12,000 \text{ cups}$$

The average coffee consumption over a specific age range using the formula:

In[39]:=
```
b0 = 150; b1 = 5; b2 = -20;

CoffeeConsumption[A_, S_] := b0 + b1 * A + b2 * S

avgValue = (1 / (50 - 30)) * NIntegrate[CoffeeConsumption[A, 6], {A, 30, 50}]

averagePlot = Plot[CoffeeConsumption[A, 6], {A, 30, 50},
   PlotLabel → "Average Coffee Consumption (Age 30 to 50)",
   AxesLabel → {"Age", "Coffee Consumption"}, Epilog →
    {Red, Dashed, Line[{{30, avgValue}, {50, avgValue}}]}, PlotStyle → Blue];
averagePlot
```

Out[41]=
```
230.
```

Out[43]=

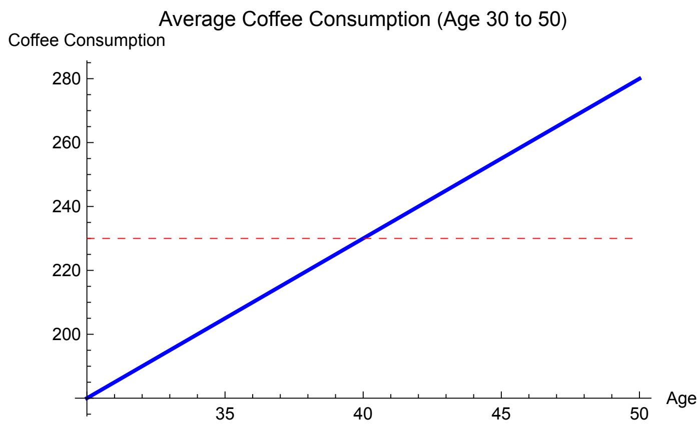

Figure 3. Average Coffee Consumption Ages 30 to 50.

The average coffee consumption graph (Figure 3) shows the average number of cups consumed per year between the ages of 30 and 50. The horizontal red dashed line at 230 cups represents this average value, calculated as:

$$\text{Average} = \frac{1}{50 - 30} \int_{30}^{50} (150 + 5\,A - 20 * 6)\,d\,A = 230\ \text{cups/year}$$

The blue line is the actual coffee consumption as a function of age, and the red dashed line is the constant average. This means that while coffee consumption increases with age (blue line), the average consumption between these ages is 230 cups/year. The calculation also confirms that, on average, a person in this age range consumes 230 cups per year.

From figures 2 and 3, we can understand that coffee consumption generally increases with age in this model, as shown by the accumulation graph and the increasing line in the average graph. The average value of 230 cups per year between ages 30 and 50 is a useful summary of typical consumption within this range. The total of 12,000 cups over a lifetime provides a clear sense of the full impact of coffee consumption for this individual.

Average value of coffee consumption

The average value of a function over an interval [a,b] is defined as:

$$\text{Average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

For our model, the average value of the function C(a,s) between ages 30 and 50 represents the typical annual coffee consumption for an individual within that age range. Calculating the average value is crucial because it simplifies the variable pattern of consumption into a single, understandable number. Instead of focusing on how consumption changes each year, the average value tells us that an individual in this age range consumes approximately 230 cups per year, even though their actual consumption may vary. This is especially useful for identifying general patterns and making comparisons. For instance, we can compare the average consumption between different age ranges (like 30-50 vs. 50-70) to understand when people tend to consume more or less coffee. The calculation of this average value using the integral divides the total coffee consumed between ages 30 and 50 by the length of that interval, effectively smoothing out the fluctuations to show a clear, consistent measure of typical consumption.

Limitations

This coffee consumption model has some limitations that should be considered. First, it is based on a simple linear function, which may not accurately capture the true relationship between age, sleep, and coffee consumption. Real-world coffee consumption might be influenced by a many other factors, including cultural habits, lifestyle changes, health concerns, stress, etc. My model's reliance only on age and sleep may overlook these other factors.

Second, the model assumes a constant relationship between sleep and coffee consumption ($b_2 = -20$), implying that each additional hour of sleep consistently reduces coffee consumption. However, this relationship may not be linear in practice. For example, a person who sleeps 4 hours might consume significantly more coffee than someone who sleeps 6 hours, but the difference between 6 and 8 hours might be minimal.

Finally, the use of Riemann sums and integrals assumes a smooth, continuous approximation of coffee consumption over time. While this is mathematically convenient, it may not fully capture the variability in an individual's actual consumption patterns, which are likely to have some more errors.

These limitations suggest that while the model provides useful insights, its results should be interpreted with caution and could be improved with more complex and realistic modeling approaches.

Conclusions

This analysis of coffee consumption using integrals provides a clear understanding of how consumption accumulates over time and how it can be summarized over specific periods using average values. The accumulated consumption graph highlights that coffee consumption increases consistently over a lifetime, leading to a substantial total of around 12,000 cups between ages 18 and 70. This result emphasizes the long-term impact of daily habits, even those that seem small.

The average value calculation is equally insightful, as it allows us to understand typical consumption without being distracted by yearly fluctuations. Knowing that the average individual consumes 230 cups per year between ages 30 and 50 helps identify this range as a period of moderate but steady coffee consumption. These tools (accumulation and average value) are useful because they turn a complex, changing function into clear, interpretable values that capture the essential patterns of coffee consumption.